Getting a System of 1st-Order Differential Equations

In general:

We have one nth order differential equation:

$$x^{(n)} = f(t, x, x', \dots, x^{(n-1)})$$

We want to get a system of only 1st order differential equations.
So we define the new variables:

$$\begin{aligned} x_1 &= x \\ x_2 &= x' \\ x_3 &= x'' \\ &\dots \\ x_n &= x^{(n-1)} \end{aligned}$$

Using these new variables we get the following differential equations (which are all 1st order):

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= x_4 \\ &\dots \\ x_{n-1}' &= x_n \\ x_n' = x^{(n)} &= f(t, x_1, x_2, \dots, x_n) \end{aligned}$$

Note: If we start off with more than one differential equation, we apply this method to each of those differential equations (if they have order 2 or more) to get a new system where each of its differential equations has order 1.

Example:

Transform $x^{(3)} + 3x'' + 2x' - 5x = \sin 2t$ into a system of 1st order differential equations.
Recall here that $x(t)$ is the function and t is the variable.

Step 1: solve for $x^{(3)}$

$$x^{(3)} = f(t, x, x', x'') = 5x - 2x' - 3x'' + \sin 2t$$

Step 2: define our new variables

$$\begin{aligned} x_1 &= x \\ x_2 &= x' \\ x_3 &= x'' \end{aligned}$$

Step 3: get our 1st order differential equations (what we were trying to find)

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' = x^{(3)} &= f(t, x_1, x_2, x_3) = 5x_1 - 2x_2 - 3x_3 + \sin 2t \end{aligned}$$